Pathfinding Algorithms

CS 351: Analysis of Algorithms

Lucas P. Cordova, Ph.D.

This lecture reviews the three pathfinding algorithms we'll use in Project 2: Greedy Best-First Search, Dijkstra's Algorithm, and A*.

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1 Introduction

1.1 What Are Pathfinding Algorithms?

Pathfinding algorithms find the optimal or near-optimal route between two points in a graph.

Applications:

- GPS navigation and route planning
- Video game AI and character movement
- Network routing protocols
- Robotics path planning

- Logistics and delivery optimization

1.2 Why Multiple Algorithms?

Different algorithms make different trade-offs:

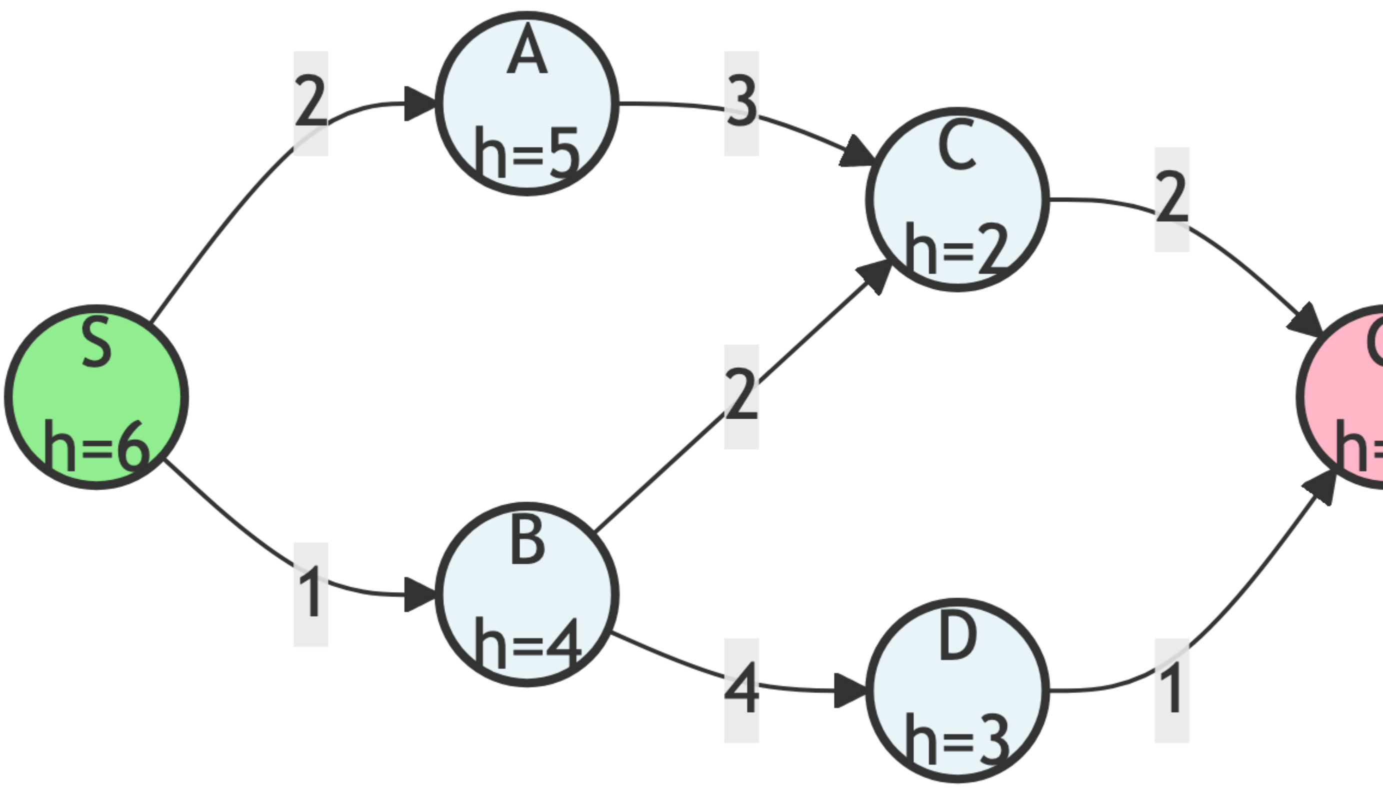
- **Speed vs. Optimality** - Fast approximation vs. guaranteed shortest path
- **Memory usage** - How much information needs to be stored
- **Use of heuristics** - Leveraging domain knowledge for efficiency

Today we'll review the three fundamental approaches to pathfinding on the same problem.

2 The Problem Setup

2.1 Our Example Graph

We'll use a simple graph with 6 nodes to illustrate each algorithm:



Goal: Find a path from S (start) to G (goal)

2.2 Graph Components

Nodes:

- S (Start) - our starting point
- A, B, C, D - intermediate nodes
- G (Goal) - our destination

Edge Costs:

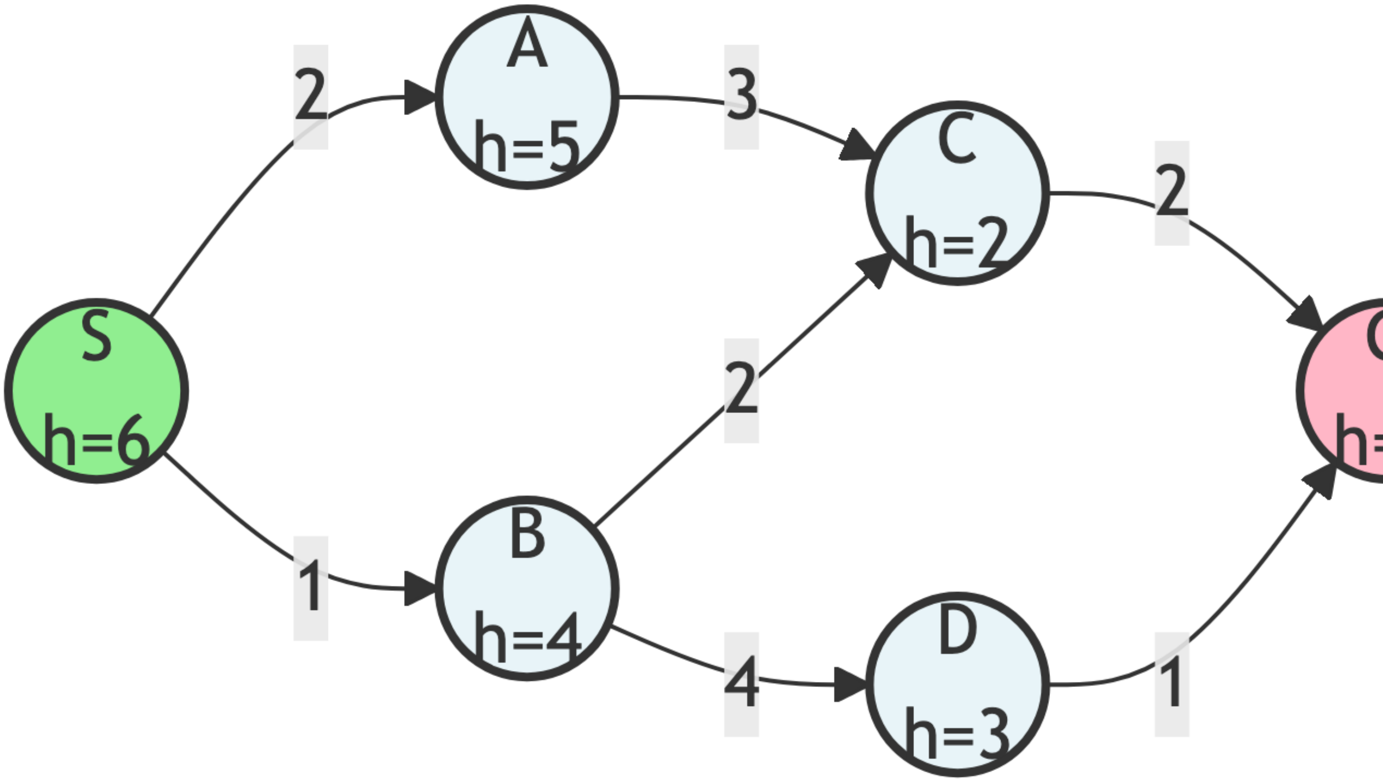
- The numbers on edges represent the actual cost to traverse the edge

Heuristic Values h(n):

- Estimated distance from each node to goal (shown below node names)
- This is a heuristic, an estimate of the remaining distance to the goal

2.3 Graph Data Table

Edge	Cost	Node	h(n)
S $\rightarrow$ A	2	S	6
S $\rightarrow$ B	1	A	5
A $\rightarrow$ C	3	B	4
B $\rightarrow$ C	2	C	2
B $\rightarrow$ D	4	D	3
C $\rightarrow$ G	2	G	0
D $\rightarrow$ G	1		



Note: The heuristic h(n) represents an estimate of the remaining distance to the goal.

3 Greedy Best-First Search

3.1 Algorithm Overview

Strategy:

- Always expand the node that appears closest to the goal based on the heuristic function h(n).

Key Characteristics:

- Uses only the heuristic h(n), ignoring actual path cost
- Greedy approach - makes locally optimal choices
- Fast but not guaranteed to find the optimal path
- Can be misled by poor heuristics

3.2 Greedy BFS: The Idea

Think of it like:

- Following the "scent" directly toward the goal, always choosing the path that seems to lead most directly there.

Analogy:

- Like a person lost in a forest who always walks in the direction that feels closest to home, without considering how difficult the terrain might be.

3.3 Greedy BFS Pseudocode

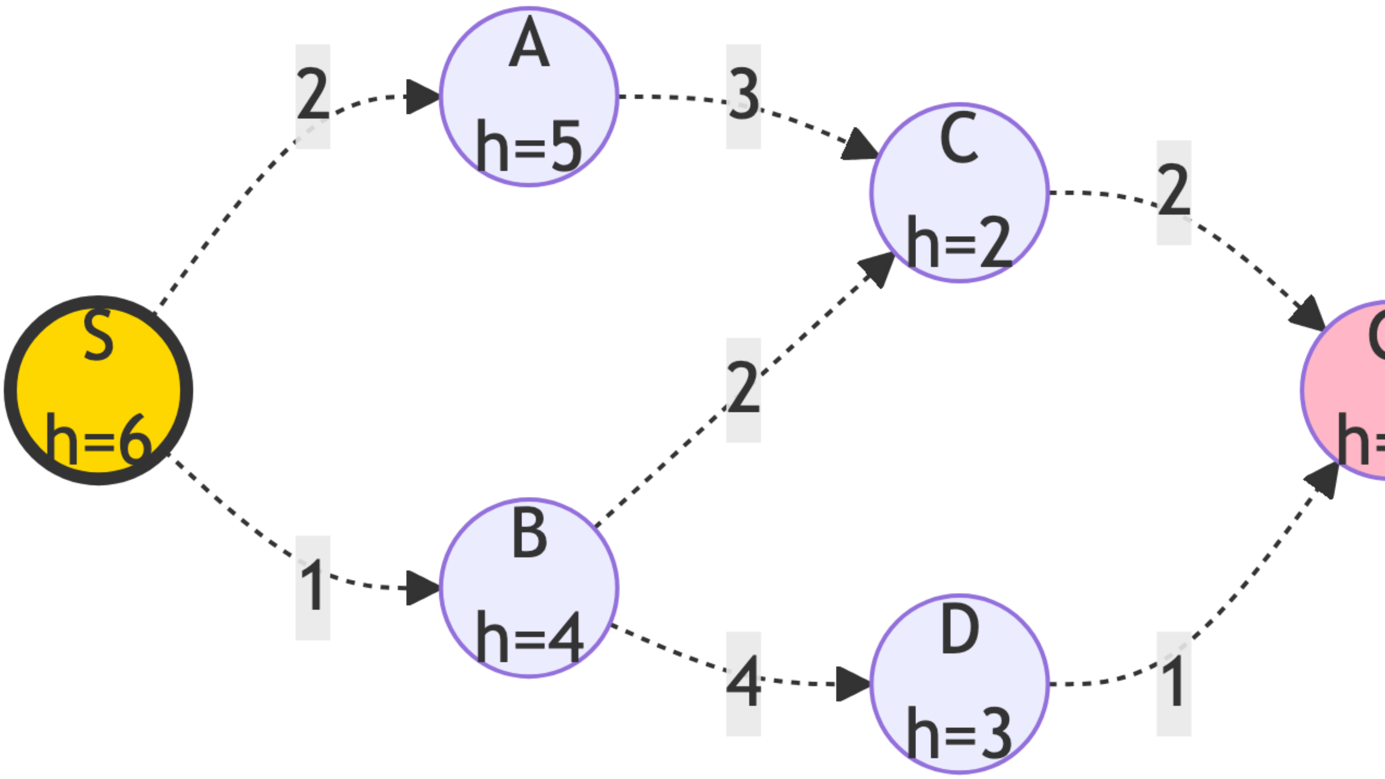
```
function GreedyBestFirstSearch(start, goal):
    frontier = PriorityQueue()
    frontier.add(start, h(start))
    explored = empty set

    while frontier is not empty:
        current = frontier.pop()  # Lowest h(n)

        if current == goal:
            return path

        explored.add(current)

        for each neighbor of current:
            if neighbor not in explored:
                frontier.add(neighbor, h(neighbor))

    return failure
```

3.4 Step 0: Initialize

Starting State:

- Frontier: {S (h=6)}
- Explored: {}
- Current path: None

Current node: S (highlighted in gold)



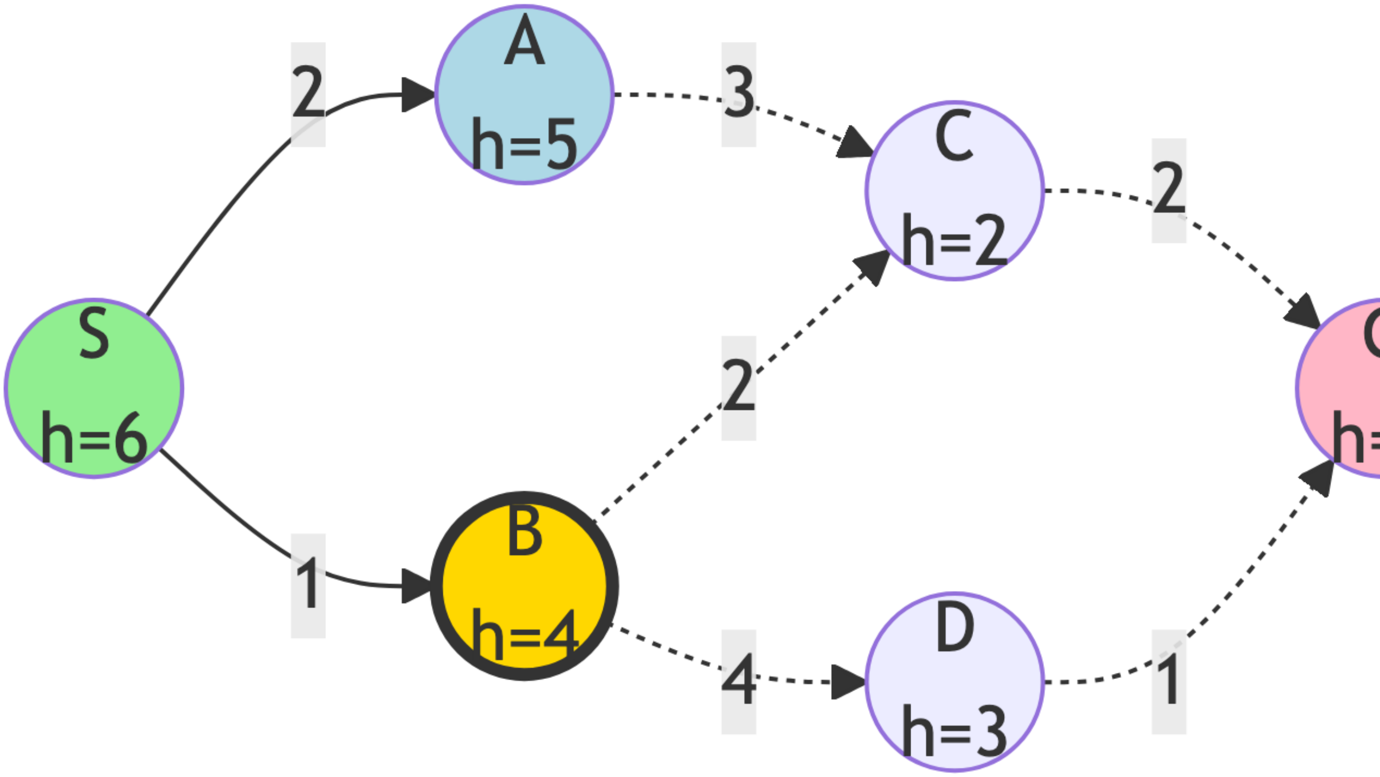
3.5 Step 1: Expand S

Action:

- Explore neighbors of S

Updates:

- Frontier: {B (h=4), A (h=5)}
- Explored: {S}
- B has lower h-value, so it will be expanded next



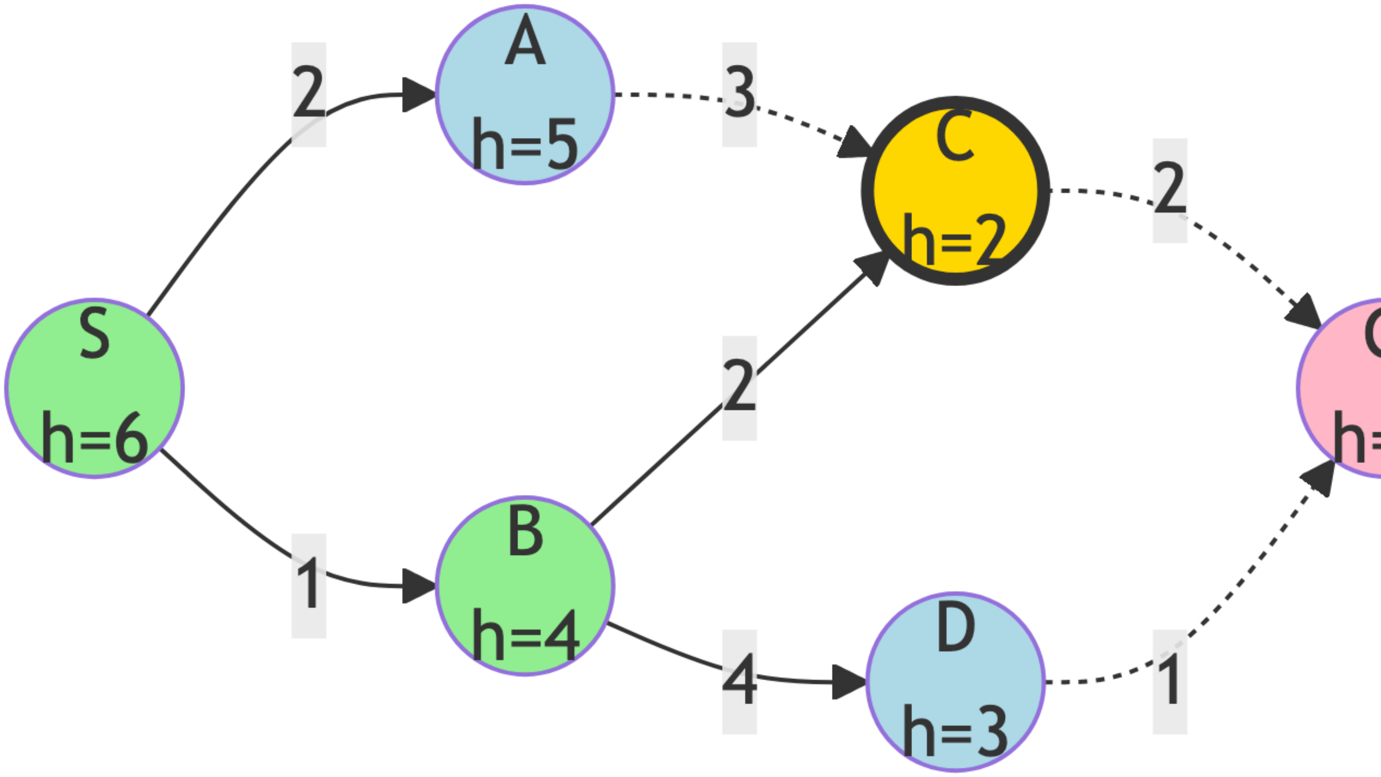
3.6 Step 2: Expand B

Action:

- B has the lowest heuristic (h=4)

Updates:

- Frontier: {C (h=2), D (h=3), A (h=5)}
- Explored: {S, B}
- C has the lowest h-value



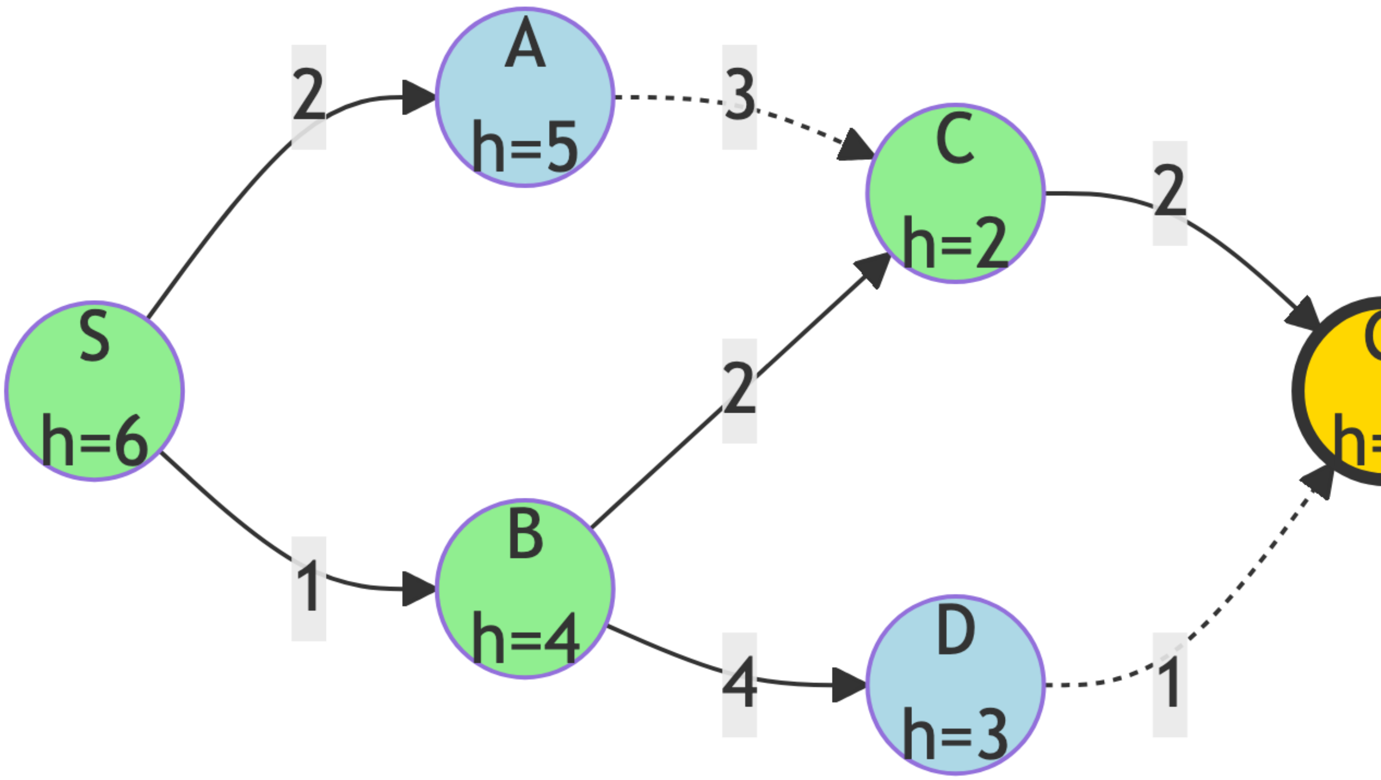
3.7 Step 3: Expand C

Action:

C has the lowest heuristic (h=2)

Updates:

- Frontier: {G (h=0), D (h=3), A (h=5)}
- Explored: {S, B, C}
- G is now in frontier!



3.8 Step 4: Reach Goal

Result:

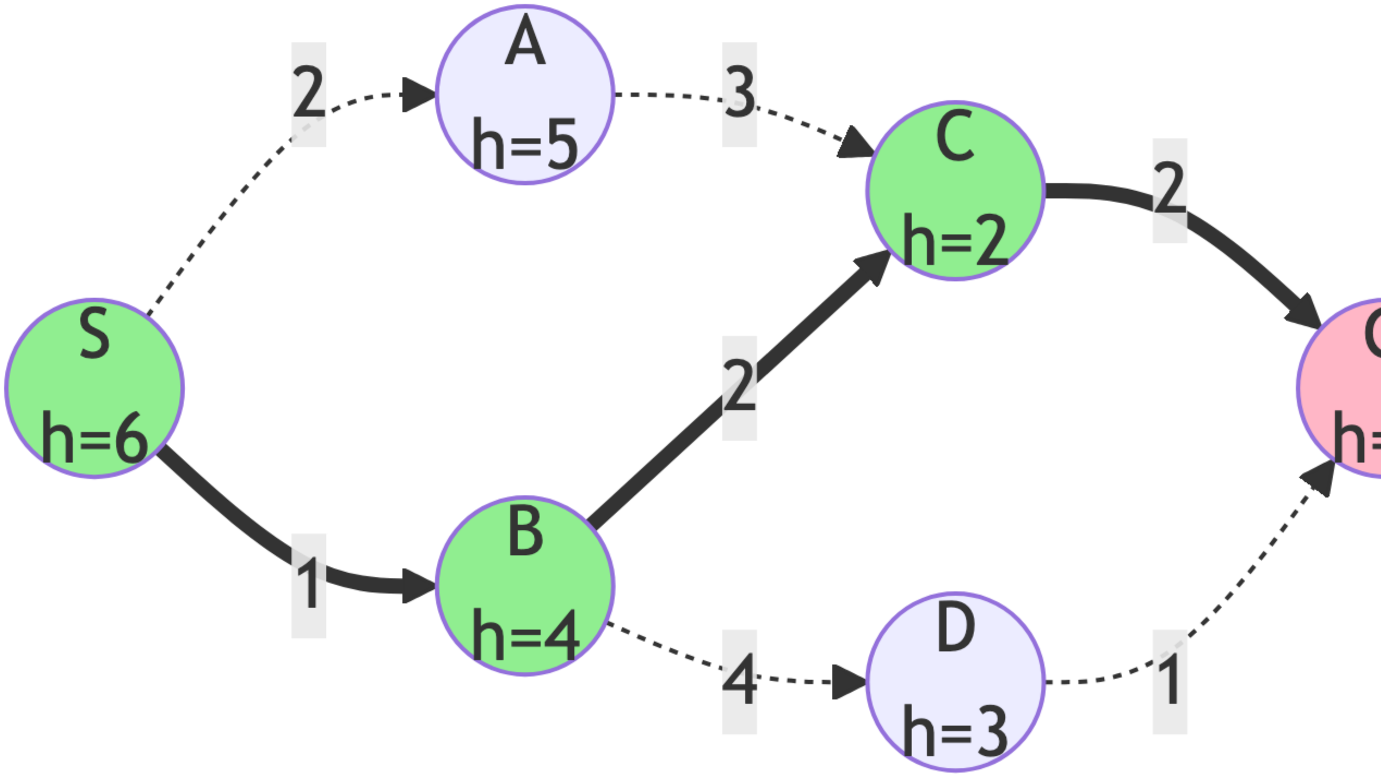
- G has h=0 (lowest possible), expand it and find the goal!

Path Found:

- S → B → C → G

Path Cost:

- 1 + 2 + 2 = 5



3.9 Greedy BFS Results

Summary:

- **Path:** S → B → C → G
- **Cost:** 5
- **Nodes Explored:** 4 (S, B, C, G)
- **Optimal?** We'll see...

Observation:

- The algorithm followed the heuristic values greedily, always choosing the node that seemed closest to the goal.

4 Dijkstra's Algorithm

4.1 Algorithm Overview

Strategy:

- Expand nodes in order of their actual distance g(n) from the start, guaranteeing the shortest path.

Key Characteristics:

- Uses actual path cost g(n), ignoring heuristics
- Guarantees optimal solution
- More thorough exploration than Greedy BFS
- Uniform cost search variant

4.2 Dijkstra's Algorithm: The Idea

Think of it like:

- Exploring all paths systematically by distance, like ripples expanding in water.

Analogy:

- Like carefully measuring every possible route with a measuring tape, always extending the shortest path found so far.

4.3 Dijkstra Pseudocode

```
function Dijkstra(start, goal):
    frontier = PriorityQueue()
    frontier.add(start, 0)
    g_scores = {start: 0}
    explored = empty set

    while frontier is not empty:
        current = frontier.pop()  # Lowest g(n)

        if current == goal:
            return path

        explored.add(current)

```

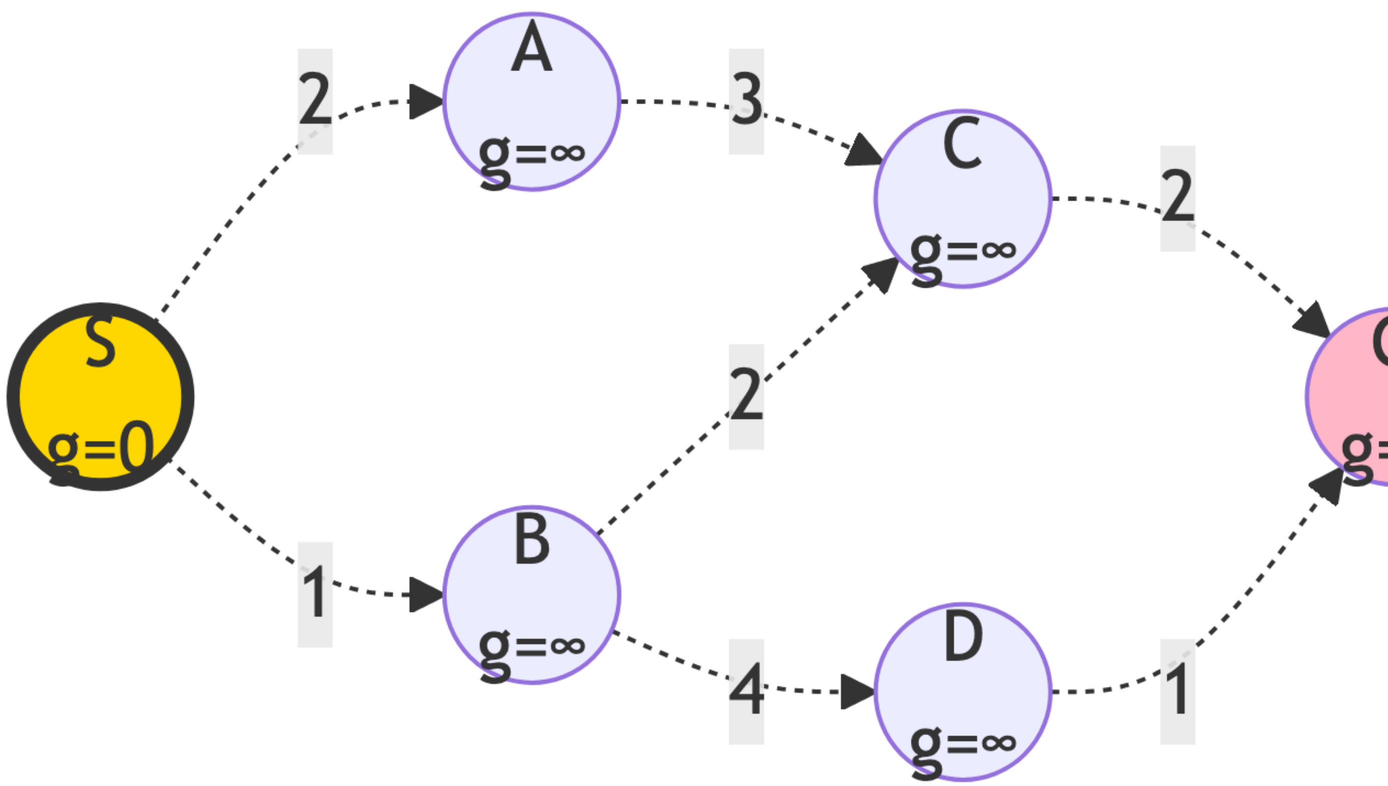
```
    for each neighbor of current with cost:
        tentative_g = g[current] + cost
        if neighbor not in explored or
           tentative_g < g[neighbor]:
            g[neighbor] = tentative_g
            frontier.add(neighbor, tentative_g)

return failure
```

4.4 Step 0: Initialize

Starting State:

- Frontier: {S (g=0)}
- g-scores: {S: 0}
- Explored: {}

All nodes start with g=∞ except start



4.5 Step 1: Expand S

Action:

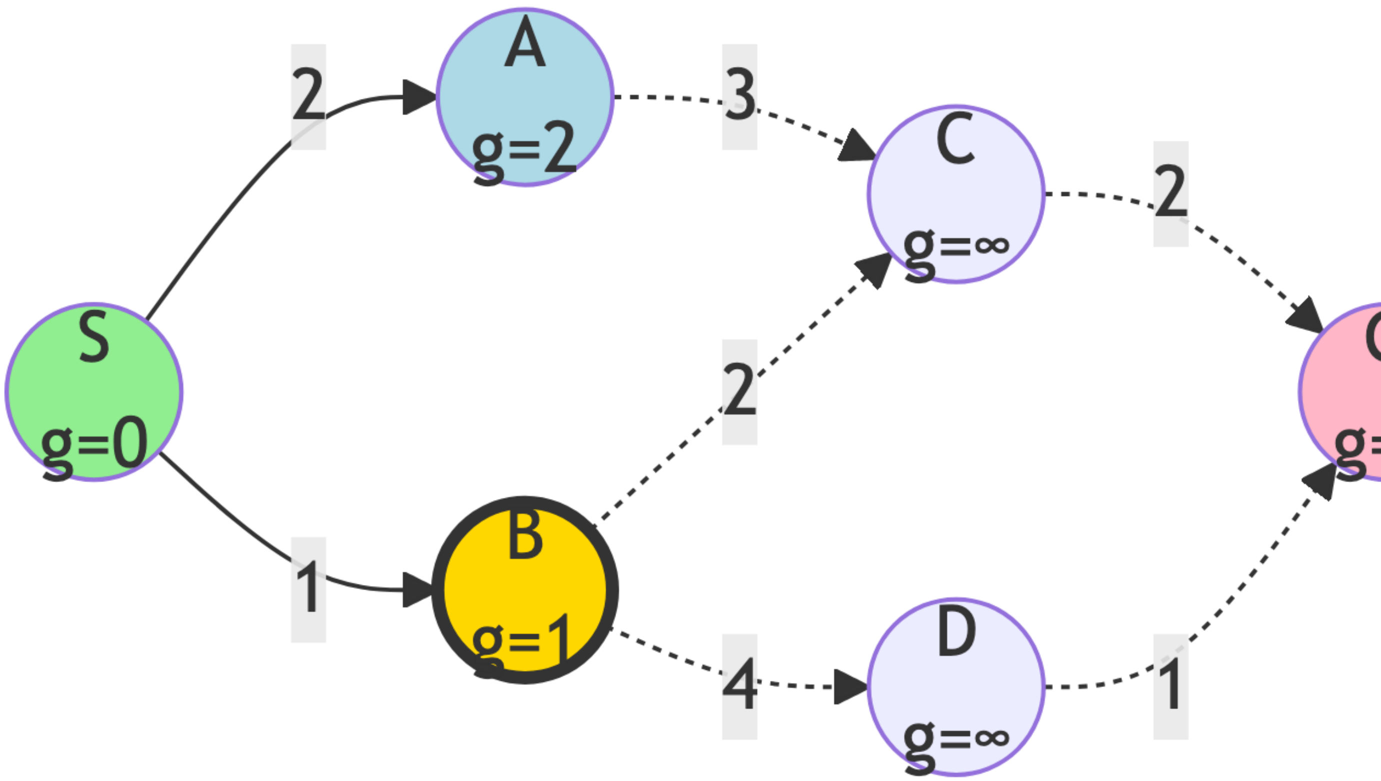
- Explore S and update neighbors

g-score Updates:

- g(A) = 0 + 2 = 2
- g(B) = 0 + 1 = 1

New State:

- Frontier: {B (g=1), A (g=2)}
- Explored: {S}



4.6 Step 2: Expand B

Action:

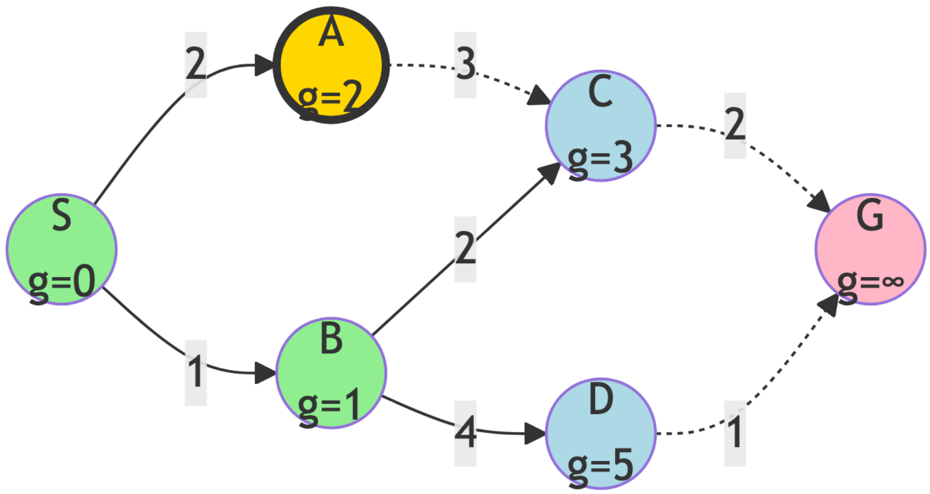
- B has lowest g-score (g=1)

g-score Updates:

- g(C) = 1 + 2 = 3
- g(D) = 1 + 4 = 5

New State:

- Frontier: {A (g=2), C (g=3), D (g=5)}
- Explored: {S, B}



4.7 Step 3: Expand A

Action:

- A has lowest g-score (g=2)

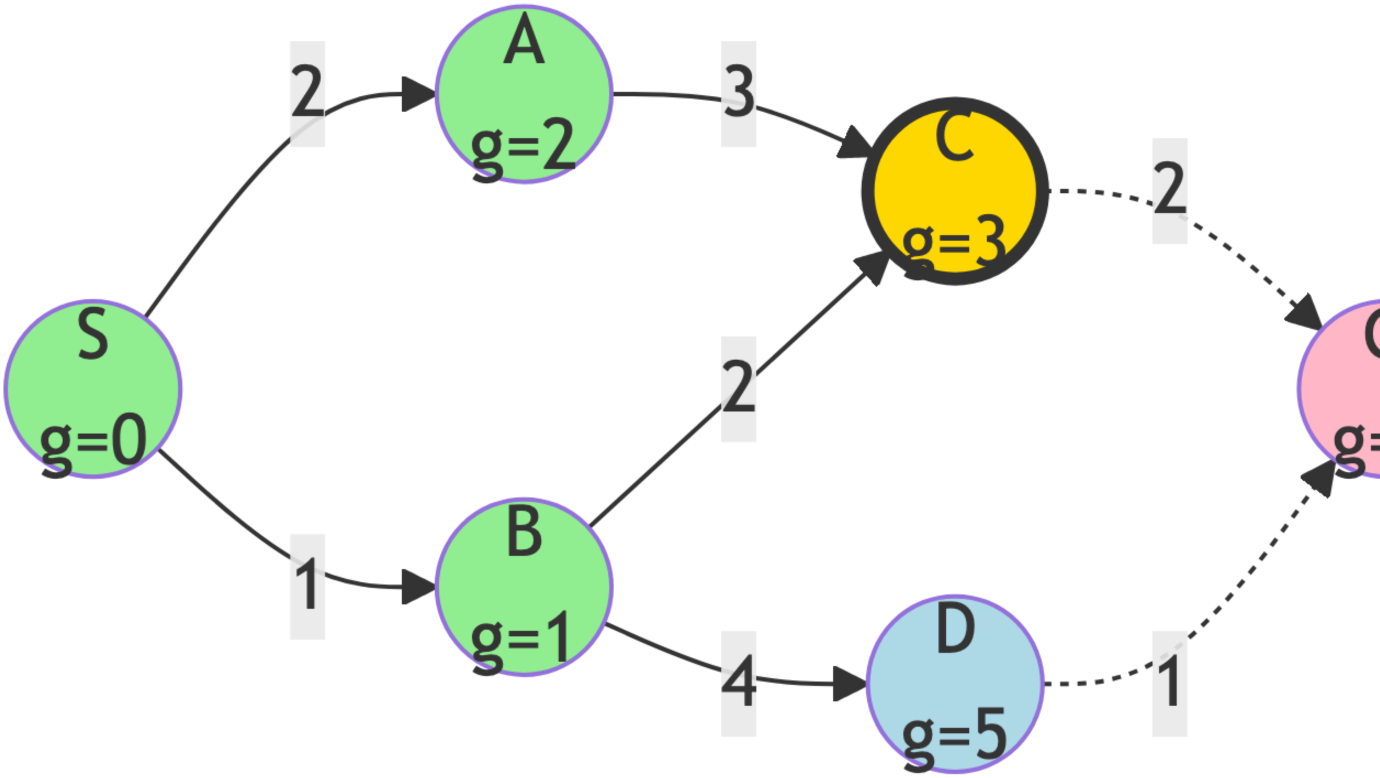
g-score Updates:

- g(C) = 2 + 3 = 5 (no update, 3 is better)

New State:

- Frontier: {C (g=3), D (g=5)}

- Explored: {S, B, A}



4.8 Step 4: Expand C

Action:

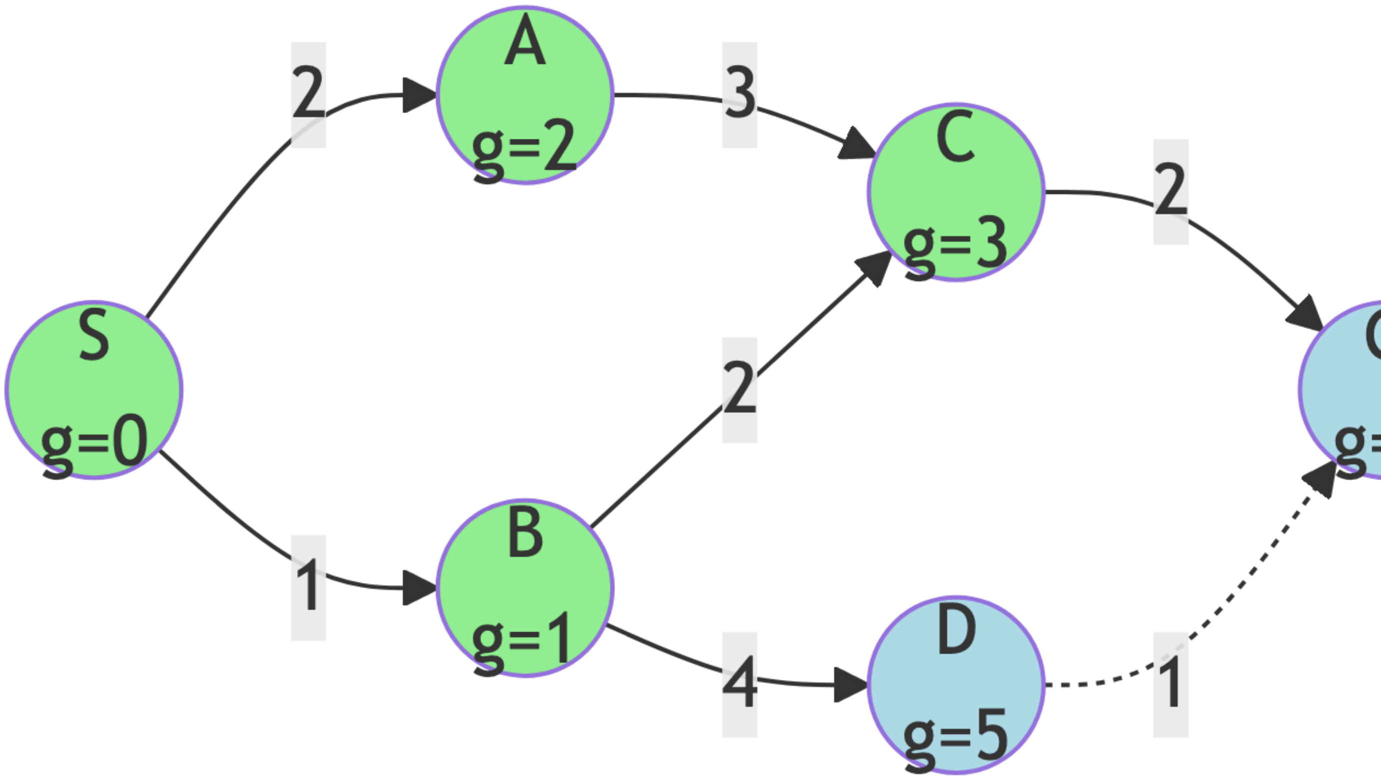
- C has lowest g-score (g=3)

g-score Updates:

- g(G) = 3 + 2 = 5

New State:

- Frontier: {D (g=5), G (g=5)}
- Explored: {S, B, A, C}
- Tie between D and G!



4.9 Step 5: Expand D (or G)

Action:

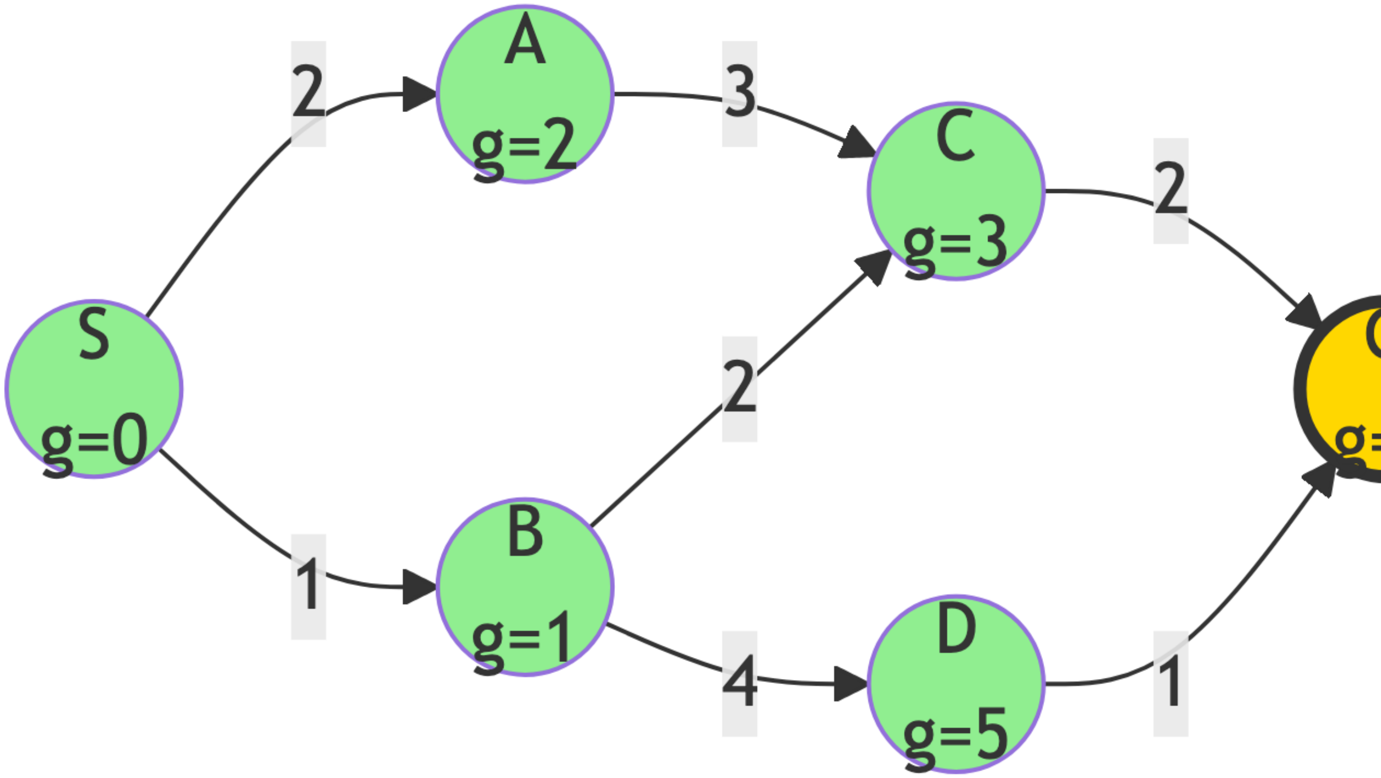
- Break tie arbitrarily - let's expand D first

g-score Updates:

- g(G) = 5 + 1 = 6 (no update, 5 is better!)

New State:

- Frontier: {G (g=5)}
- Explored: {S, B, A, C, D}



4.10 Step 6: Reach Goal

Result:

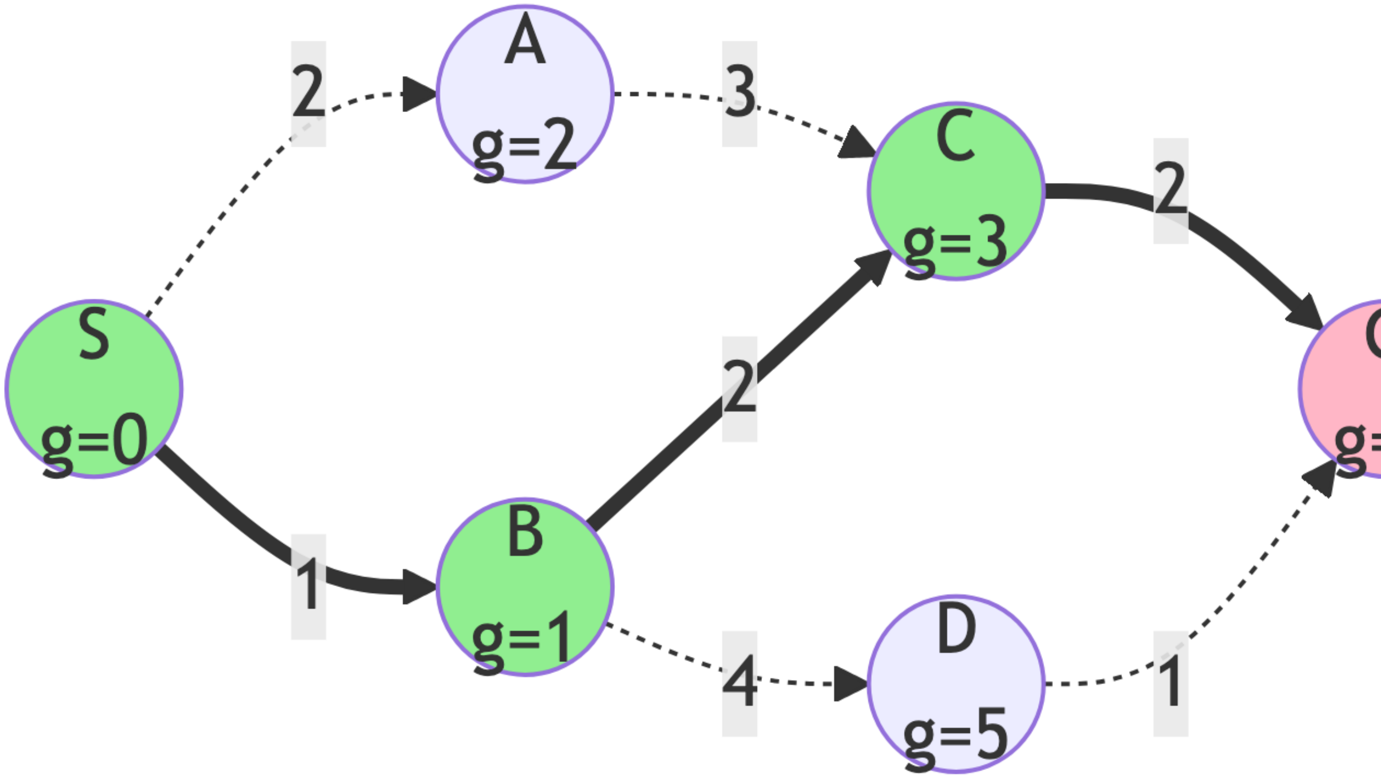
- G is expanded - goal reached!

Path Found:

- S → B → C → G

Path Cost:

1 + 2 + 2 = 5



4.11 Dijkstra Results

Summary:

- **Path:** S → B → C → G
- **Cost:** 5
- **Nodes Explored:** 6 (S, B, A, C, D, G)
- **Optimal?** YES - guaranteed!

Observation:

- Dijkstra explored more nodes than Greedy BFS but found the same path. The key difference: Dijkstra guarantees this is optimal.

4.12 Why Is This Path Optimal?

Alternative Path Analysis:

- S → A → C → G = 2 + 3 + 2 = 7 (worse)
- S → B → D → G = 1 + 4 + 1 = 6 (worse)
- S → B → C → G = 1 + 2 + 2 = 5 (best!)

Dijkstra checked all possibilities systematically and confirmed 5 is the minimum cost.

5 A* Search Algorithm

5.1 Algorithm Overview

Strategy:

- Combine actual cost g(n) and heuristic h(n) using evaluation function f(n) = g(n) + h(n)

Key Characteristics:

- Uses both actual cost and heuristic information
- Optimal when heuristic is admissible (never overestimates)
- More efficient than Dijkstra with good heuristics
- Best of both worlds

5.2 A* Search: The Idea

Think of it like:

- A smart explorer who considers both how far they've traveled AND how far they estimate they still need to go.

Analogy:

- Like planning a road trip where you consider both the miles already driven and your GPS estimate of remaining distance.

5.3 A* Pseudocode

```
function AStar(start, goal):
    frontier = PriorityQueue()
    frontier.add(start, h(start))
    g_scores = {start: 0}
    explored = empty set

    while frontier is not empty:
        current = frontier.pop()  # Lowest f(n)

        if current == goal:
            return path

        explored.add(current)

        for each neighbor of current with cost:
            tentative_g = g[current] + cost
            if neighbor not in explored or
               tentative_g < g[neighbor]:
                g[neighbor] = tentative_g
                f[neighbor] = g[neighbor] + h[neighbor]
                frontier.add(neighbor, f[neighbor])

    return failure
```

5.4 The f(n) Function

Evaluation Function:

- f(n) = g(n) + h(n)

Components:

- **g(n)** = actual cost from start to node n
- **h(n)** = estimated cost from node n to goal
- **f(n)** = estimated total cost of path through n

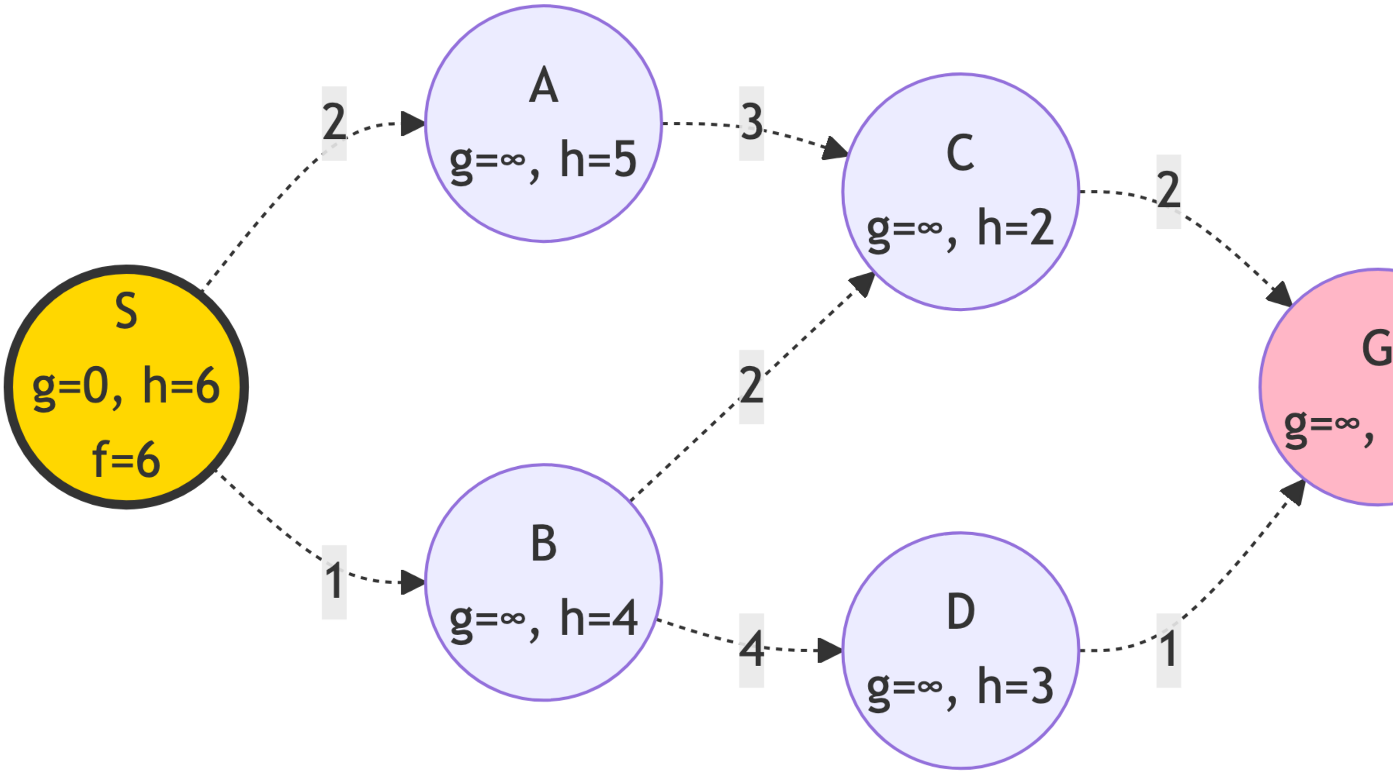
Intuition:

- We want to explore paths that have both low actual cost so far AND promise to reach the goal efficiently.

5.5 Step 0: Initialize

Starting State:

- Frontier: {S (f=0+6=6)}
- g-scores: {S: 0}
- Explored: {}



5.6 Step 1: Expand S

Action:

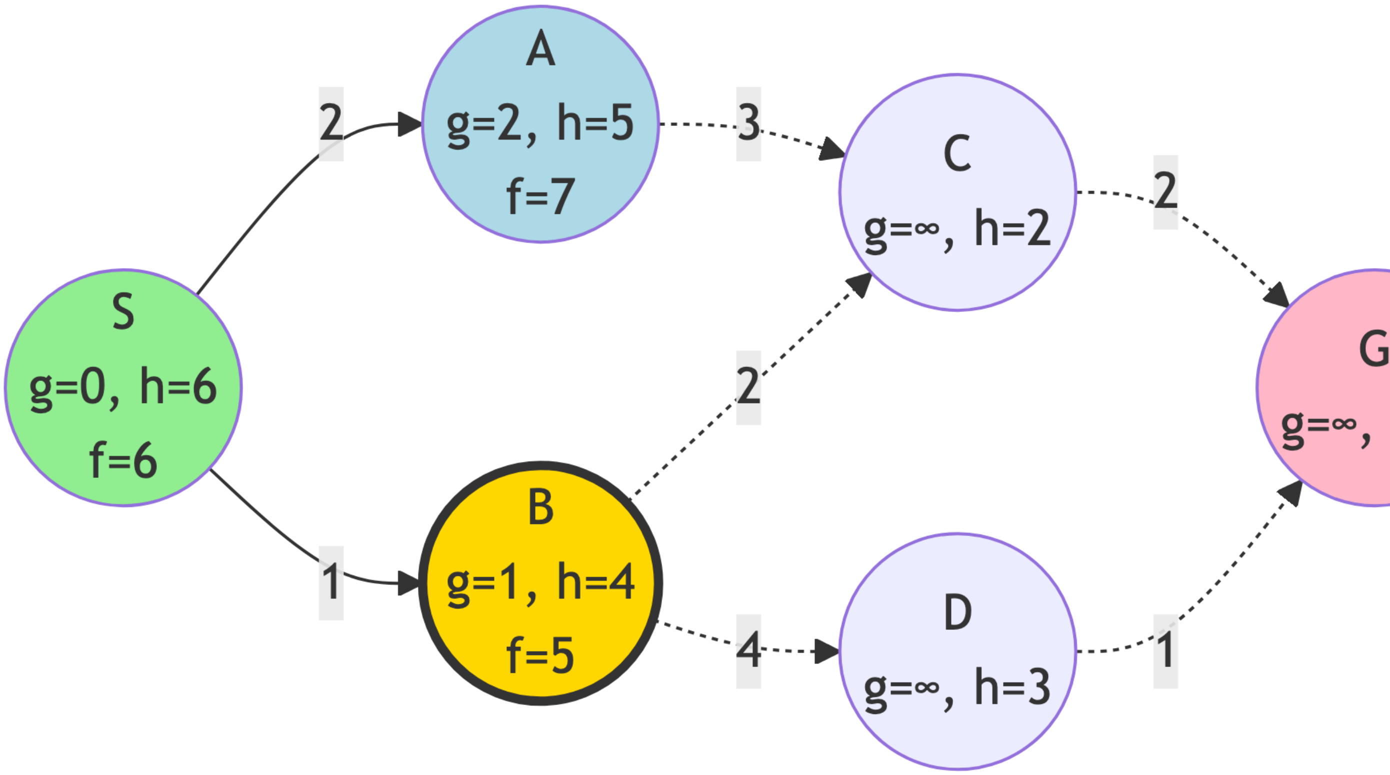
- Calculate f-scores for neighbors

f-score Updates:

- A: g=2, h=5, f=7
- B: g=1, h=4, f=5

New State:

- Frontier: {B (f=5), A (f=7)}
- Explored: {S}



5.7 Step 2: Expand B

Action:

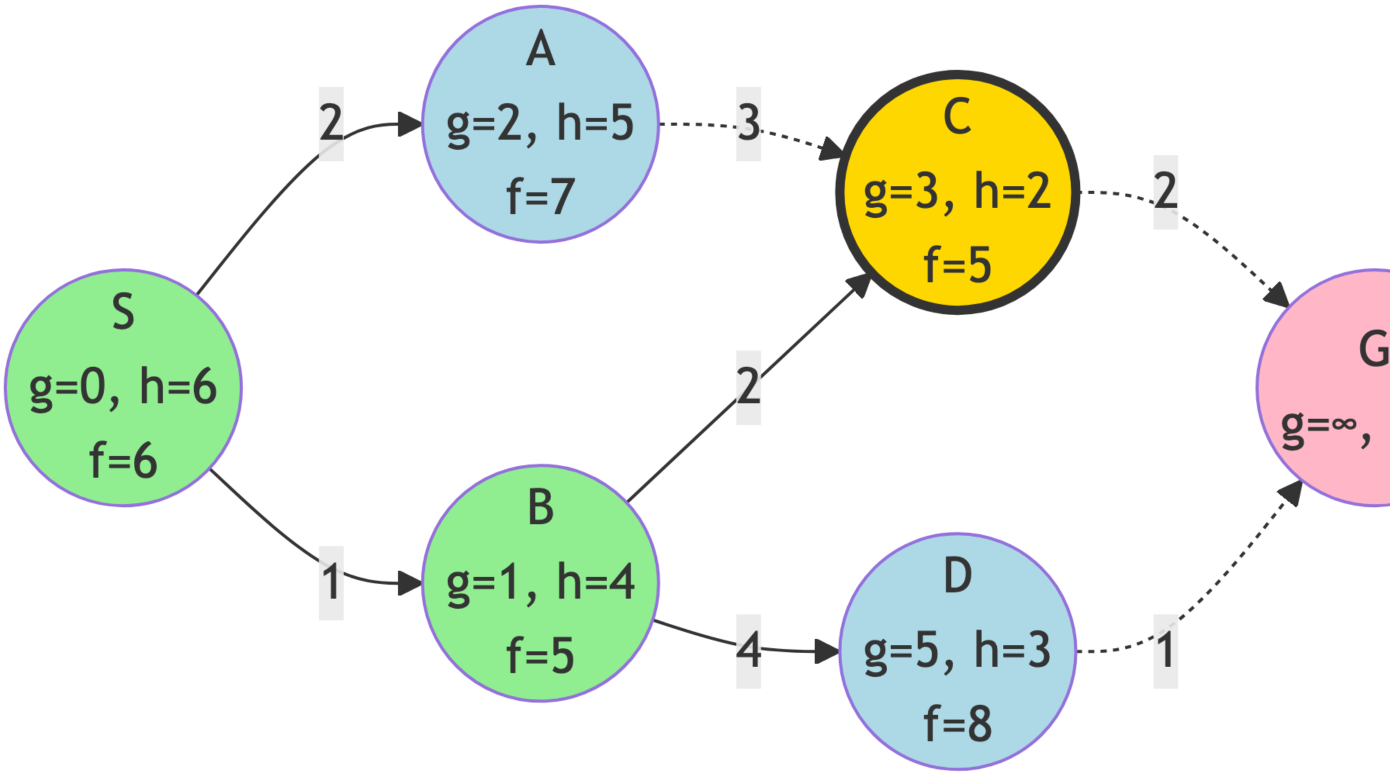
- B has lowest f-score (f=5)

f-score Updates:

- C: g=3, h=2, f=5
- D: g=5, h=3, f=8

New State:

- Frontier: {C (f=5), A (f=7), D (f=8)}
- Explored: {S, B}



5.8 Step 3: Expand C

Action:

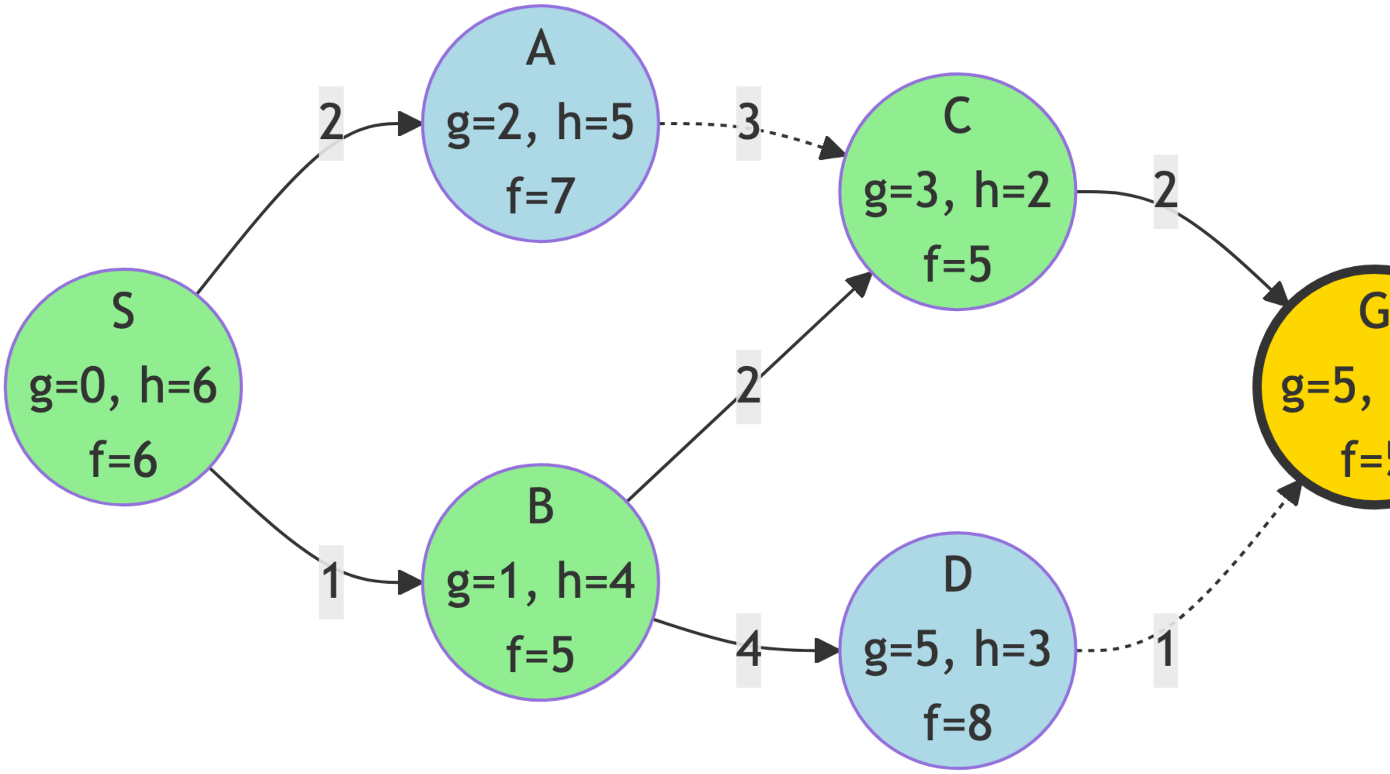
- C has lowest f-score (f=5)

f-score Updates:

- G: g=5, h=0, f=5

New State:

- Frontier: {G (f=5), A (f=7), D (f=8)}
- Explored: {S, B, C}



5.9 Step 4: Reach Goal

Result:

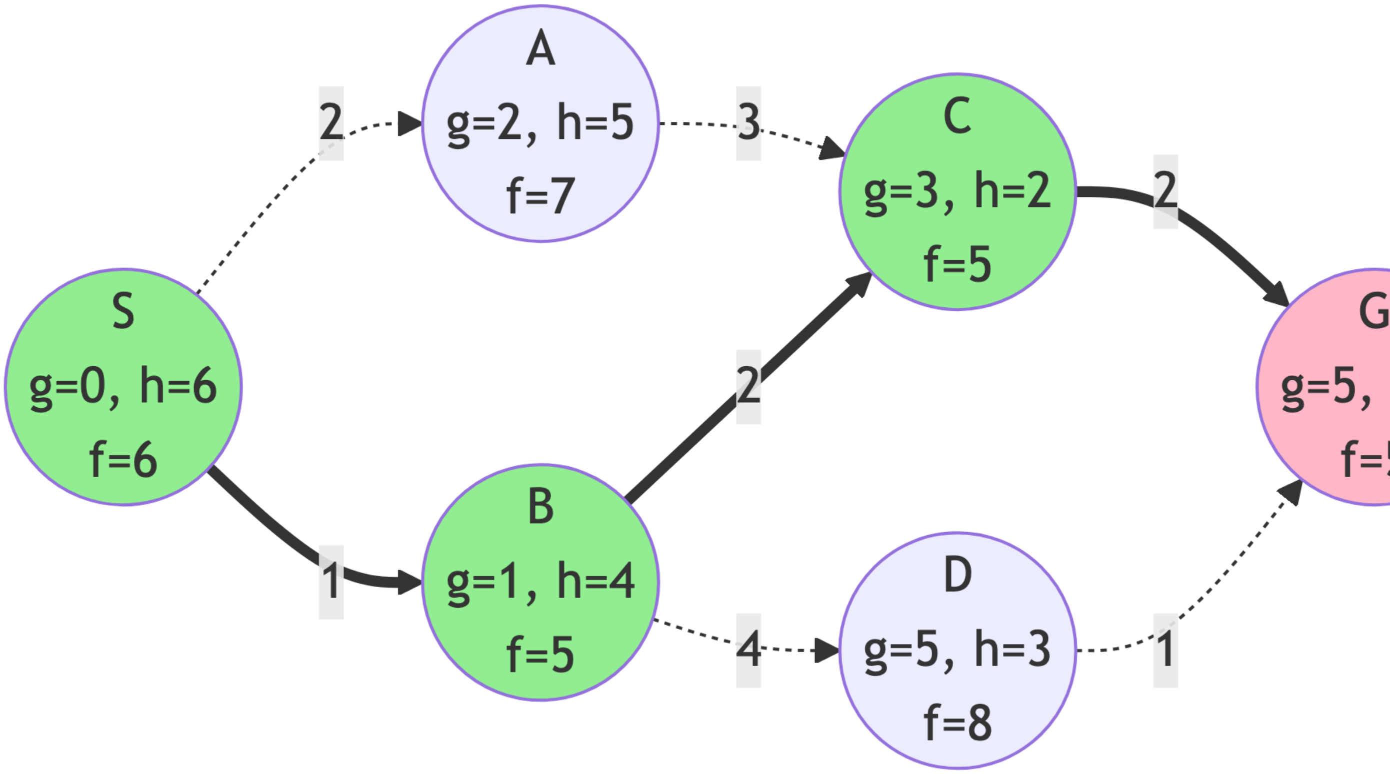
- G has lowest f-score (f=5) - goal reached!

Path Found:

- S → B → C → G

Path Cost:

- $1 + 2 + 2 = 5$



5.10 A* Results

Summary:

- **Path:** S → B → C → G
- **Cost:** 5
- **Nodes Explored:** 4 (S, B, C, G)
- **Optimal?** YES (with admissible heuristic)

Observation:

- A* explored the same number of nodes as Greedy BFS (4) but with the optimality guarantee of Dijkstra!

5.11 Why A* Was Efficient

The Power of f(n):

- A* avoided exploring A and D because their f-scores indicated they wouldn't lead to better solutions.

Comparison:

- Node A: f=7 (higher than solution)
- Node D: f=8 (higher than solution)
- Solution path: all nodes had f 5

This is why A* is often the best choice when a good heuristic is available.

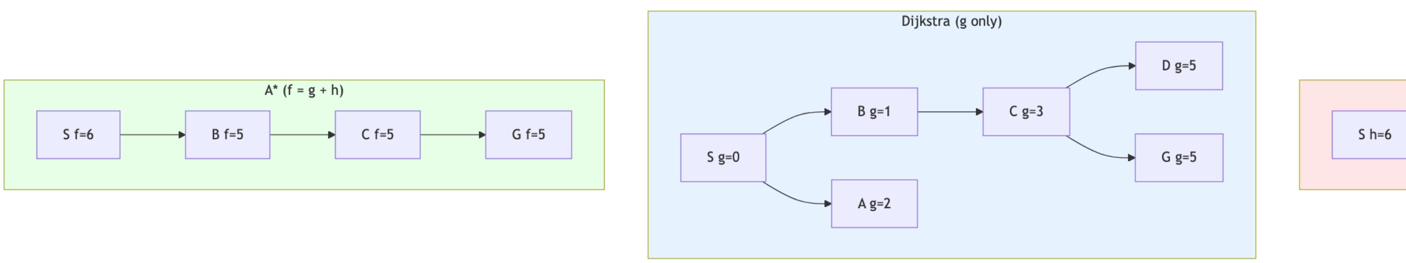
6 Algorithm Comparison

6.1 Side-by-Side Results

Algorithm	Path	Cost	Nodes Explored	Optimal?
Greedy BFS	S→B→C→G	5	4	Yes*
Dijkstra	S→B→C→G	5	6	Yes
A*	S→B→C→G	5	4	Yes

*Greedy BFS happened to find the optimal path but doesn't guarantee it

6.2 Exploration Pattern Comparison



6.3 Performance Metrics

Time Complexity:

- All three: O((V + E) log V) with priority queue
- In practice, efficiency varies with heuristic quality

Space Complexity:

- All three: O(V) for storing nodes
- A* may need more memory for f-scores

Optimality:

- Greedy BFS: No guarantee
- Dijkstra: Always optimal
- A*: Optimal with admissible heuristic

6.4 When to Use Each Algorithm

Greedy Best-First Search:

- When speed is critical and approximate solutions are acceptable
- When you have a very accurate heuristic
- Video games, real-time systems

Dijkstra's Algorithm:

- When optimality is required and no heuristic is available
- When all edges have different costs
- Network routing, GPS without traffic data

A* Search:

- When optimality is required and good heuristics exist
- Most pathfinding applications
- GPS with traffic data, game AI, robotics

6.5 Heuristic Quality Matters

Admissible Heuristics:

- Never overestimate the actual cost (h(n)   actual cost)

Good Heuristics:

- Lead A* to explore fewer nodes
- Make A* more efficient than Dijkstra
- Examples: Euclidean distance, Manhattan distance

Poor Heuristics:

- If h(n) = 0 for all nodes, A* becomes Dijkstra
- If h(n) overestimates, A* may not be optimal
- Balance accuracy and computation time

7 Key Takeaways

7.1 Core Concepts

Three Different Strategies:

1. **Greedy** - Follow what looks best locally (fast, risky)
2. **Systematic** - Check everything methodically (slow, guaranteed)
3. **Informed** - Use knowledge to guide systematic search (efficient, guaranteed)

The Trade-off Triangle:

- Speed   Optimality   Information Requirements

7.2 Practical Applications

Real-World Impact:

- Google Maps uses A*-like algorithms
- Video games use optimized variants for NPC pathfinding
- Robots use these for navigation
- Network protocols use Dijkstra variants

Choosing the Right Algorithm:

- Consider your constraints and requirements before selecting an approach.